

Correction to Theorem 6.3 of Uniqueness of A_∞ -structures and Hochschild cohomology

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This is a corrected version of Theorem 6.3 of “Uniqueness of A_∞ -structures and Hochschild cohomology”. The previous version claims in the proof that the element $D(a_4)$ is trivial for degree reasons, which is simply not true. In order to fix this, a slightly stronger assumption in the statement of the theorem is needed. The corrected statement and proof are below.

Theorem Let A be a dga whose minimal model $H^*(A)$ satisfies $m_i = 0$ for $i \neq 2, 3$. If $F^4 \text{HH}_{\text{alg}}^2(H^*(A), H^*(A)) = 0$, where HH_{alg}^* denotes the Hochschild cohomology of associative algebras, then any A_∞ -structure \bar{m} on $H^*(A)$ with $\bar{m}_1 = 0$, $\bar{m}_2 = m_2$ and $\bar{m}_3 = m_3$ is quasi-isomorphic to m .

Proof The proof is extremely similar to the proof of Theorem 5.3.

The differential in the Hochschild complex for $H^*(A)$ is

$$D = D_2 = [m_2, -] : C^{n,k}(H^*(A), H^*(A)) \longrightarrow C^{n+1,k}(H^*(A), H^*(A)).$$

Assume there is an A_∞ -structure \bar{m} on $H^*(A)$ with

$$\bar{m} = m_2 + m_3 + a_4 + a_5 + \cdots .$$

Let

$$a = a_4 + a_5 + \cdots .$$

Because $m = m_2 + m_3$ is an A_∞ -structure on the minimal model by assumption, we know that a is a twisting cochain, i.e. a satisfies the Maurer-Cartan equation $-D(a) = a \circ a$. Again, for degree reasons $D(a_4) = 0$, so $[a_4]$ is a class in $F^4 \text{HH}_{\text{alg}}^2(H^*(A), H^*(A))$. However, we assumed this to be trivial, thus a_4 must also be a boundary. In other words, there is a p such that $D(p) = a_4$.

By the analogue of Lemma 5.1 for this case, there is a twisting cochain $\bar{a} = \bar{a}_4 + \bar{a}_5 + \cdots$ such that

- \bar{a} is equivalent to a ,
- $\bar{a}_k = a_k$ for $k \leq 3$,
- $\bar{a}_4 = a_4 - D(p) = 0$.

The rest of the proof continues inductively following the same steps as the proof of Theorem 5.3. □

Remark Since the Hochschild cohomology of associative algebras is bigraded, we can express the condition $F^4 \text{HH}_{alg}^2(H^*(A), H^*(A)) = 0$ as $\text{HH}_{alg}^{n, 2-n}(H^*(A), H^*(A)) = 0$ for all $n \geq 4$.

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